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Two collinear interface cracks in magneto-electro-elastic composites

Zhen-Gong Zhou *, Biao Wang, Yu-Guo Sun

Center for Composite Materials and Electro-Optics Research Center, Harbin Institute of Technology, P.O. Box 1247, Harbin 150001, PR China

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Abstract

In this paper, the behavior of two symmetric interface cracks between two dissimilar magneto-electro-elastic composite half planes under anti-plane shear stress loading is investigated by Schmidt method for the permeable crack surface conditions. By using the Fourier transform, the problem can be solved with a set of triple integral equations in which the unknown variable is the jump of the displacements across the crack surfaces. In solving the triple integral equations, the jump of the displacements across the crack surface is expanded in a series of Jacobi polynomials. Numerical solutions of the stress intensity factor are given. The relations among the electric filed, the magnetic flux and the stress field can be obtained.
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Keywords: Magneto-electro-elastic composites; Interface crack; Triple integral equations

1. Introduction

Composite material consisting of a piezoelectric phase and a piezomagnetic phase has drawn significant interest in recent years, due to the rapid development in adaptive material systems. It shows a remarkably large magnetoeletric coefficient, the coupling coefficient between static electric and magnetic fields, which does not exist in either constituent. The magnetoelectric coupling is a new product property of the composite, since it is absent in each constituent. In some cases, the coupling effect of piezoelectric/piezomagnetic composites can be even obtained a

* Corresponding author.
E-mail address: zhouzhg@hit.edu.cn (Z.-G. Zhou).

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hundred times larger than that in a single-phase magnetoelectric material. Consequently, they are extensively used as magnetic field probes, electric packaging, acoustic, hydrophones, medical ultrasonic imaging, sensors, and actuators with the responsibility of magneto-electro-mechanical energy conversion [1]. When subjected to mechanical, magnetic and electrical loads in service, these magneto-electro-elastic composites can fail prematurely due to some defects, e.g. cracks, holes, etc. arising during their manufacturing processes. Therefore, it is of great importance to study the magneto-electro-elastic interaction and fracture behaviors of magneto-electro-elastic composites [2,3].

The development of piezoelectric–piezomagnetic composites has its roots in the early work of Van Suchtelen [4] who proposed that the combination of piezoelectric–piezomagnetic phases may exhibit a new material property—the magnetoelectric coupling effect. Since then, the magneto-electric coupling effect of $BaTiO_3$–$CoFe_2O_4$ composites has been measured by many researchers. Much of the theoretical work for the investigation of magnetoelectric coupling effect has only recently been studied [1–3,5–10]. The behaviors of two collinear cracks in piezoelectric materials were studied in [11–13]. To our knowledge, the magneto-electro-elastic behavior of magneto-electro-elastic composites with two collinear symmetric interface cracks subjected to anti-plane shear stress loading has not been studied.

In this paper, the behavior of two collinear interface cracks between to dissimilar magneto-electro-elastic half planes composites subjected to anti-plane shear is investigated by use of a somewhat different method, named as the Schmidt method [14,15]. The Fourier transform is applied and a mixed boundary value problem is reduced to a triple integral equations. To solve the triple integral equations, the jump of the displacements across the crack surfaces is expanded in a series of Jacobi polynomials. This process is quite different from those adopted in the references [2,3] as mentioned above. The form of solution is easy to understand. Numerical solutions are obtained for the stress.

2. Formulation of the problem

It is assumed that there are two collinear interface cracks of length $1 - b$ between two dissimilar magneto-electro-elastic composite half planes as shown in Fig. 1. $2b$ is the distance between two cracks. The piezoelectric/piezomagnetic boundary-value problem for anti-plane shear is considerably simplified if we consider only the out-of-plane displacement, the in-plane electric and the in-plane magnetic fields. As discussed in [16], since the opening displacement is zero for the present anti-plane shear problem; the crack surfaces can be assumed to be in perfect contact. Accordingly, both the electric and magnetic potentials are assumed to be continuous across the

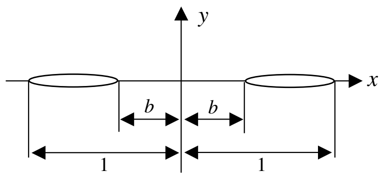

Fig. 1. Two interface cracks between two dissimilar magneto-electro-elastic composite half planes.

crack surfaces. So the boundary conditions of the present problem are (we just consider the perturbation field.):

$$\begin{cases} \tau_{yz}^{(1)}(x,0^+) = \tau_{yz}^{(2)}(x,0^-) = -\tau_0, & b \leqslant |x| \leqslant 1 \\ w^{(1)}(x,0^+) = w^{(2)}(x,0^-), & |x| < b, \ |x| > 1 \end{cases} \tag{1}$$

$$\phi^{(1)}(x,0^+) = \phi^{(2)}(x,0^-), \quad D_y^{(1)}(x,0^+) = D_y^{(2)}(x,0^-), \quad |x| \leqslant \infty \tag{2}$$

$$\psi^{(1)}(x,0^+) = \psi^{(2)}(x,0^-), \quad B_y^{(1)}(x,0^+) = B_y^{(2)}(x,0^-), \quad |x| \leqslant \infty \tag{3}$$

$$w^{(1)}(x,y) = w^{(2)}(x,y) = 0 \quad \text{for } (x^2+y^2)^{1/2} \to \infty \tag{4}$$

where $\tau_{zk}^{(i)}$, $D_k^{(i)}$ and $B_k^{(i)}$ $(k = x, y, i = 1, 2)$ are the anti-plane shear stress, in-plane electric displacement and in-plane magnetic flux, respectively. $w^{(i)}$, $\phi^{(i)}$ and $\psi^{(i)}$ are the mechanical displacement, the electric potential and the magnetic potential, respectively. Note that all quantities with superscript i $(i = 1, 2)$ refer to the upper half plane 1 and the lower half plane 2 as in Fig. 1, respectively. In this paper, we only consider that τ_0 is positive.

It is assumed that the magneto-electro-elastic composite is transversely isotropic. So the constitutive equations can be written as

$$\tau_{zk}^{(i)} = c_{44}^{(i)} w_{,k}^{(i)} + e_{15}^{(i)} \phi_{,k}^{(i)} + q_{15}^{(i)} \psi_{,k}^{(i)}, \quad (k = x, y, \ i = 1, 2) \tag{5}$$

$$D_k^{(i)} = e_{15}^{(i)} w_{,k}^{(i)} - \varepsilon_{11}^{(i)} \phi_{,k}^{(i)} - d_{11}^{(i)} \psi_{,k}^{(i)}, \quad (k = x, y, \ i = 1, 2) \tag{6}$$

$$B_k^{(i)} = q_{15}^{(i)} w_{,k}^{(i)} - d_{11}^{(i)} \phi_{,k}^{(i)} - \mu_{11}^{(i)} \psi_{,k}^{(i)}, \quad (k = x, y, \ i = 1, 2) \tag{7}$$

where $c_{44}^{(i)}$ is shear modulus, $e_{15}^{(i)}$ is piezoelectric coefficient, $\varepsilon_{11}^{(i)}$ is dielectric parameter, $q_{15}^{(i)}$ is piezomagnetic coefficient, $d_{11}^{(i)}$ is electromagnetic coefficient, $\mu_{11}^{(i)}$ is magnetic permeability.

The anti-plane governing equations are

$$c_{44}^{(i)} \nabla^2 w^{(i)} + e_{15}^{(i)} \nabla^2 \phi^{(i)} + q_{15}^{(i)} \nabla^2 \psi^{(i)} = 0, \quad (i = 1, 2) \tag{8}$$

$$e_{15}^{(i)} \nabla^2 w^{(i)} - \varepsilon_{11}^{(i)} \nabla^2 \phi^{(i)} - d_{11}^{(i)} \nabla^2 \psi^{(i)} = 0, \quad (i = 1, 2) \tag{9}$$

$$q_{15}^{(i)} \nabla^2 w^{(i)} - d_{11}^{(i)} \nabla^2 \phi^{(i)} - \mu_{11}^{(i)} \nabla^2 \psi^{(i)} = 0, \quad (i = 1, 2) \tag{10}$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the two-dimensional Laplace operator. Because of the assumed symmetry in geometry and loading, it is sufficient to consider only the problem for $0 \leqslant x < \infty$, $-\infty \leqslant y < \infty$. A Fourier transform is applied to Eqs. (8)–(10). Assumed that the solutions are

$$\begin{cases} w^{(1)}(x,y) = \frac{2}{\pi}\int_0^\infty A_1(s)\mathrm{e}^{-sy}\cos(sx)\mathrm{d}s \\ \phi^{(1)}(x,y) = \dfrac{\mu_{11}^{(1)}e_{15}^{(1)} - d_{11}^{(1)}q_{15}^{(1)}}{\varepsilon_{11}^{(1)}\mu_{11}^{(1)} - d_{11}^{(1)2}} w^{(1)}(x,y) + \dfrac{2}{\pi}\displaystyle\int_0^\infty B_1(s)\mathrm{e}^{-sy}\cos(sx)\mathrm{d}s, \quad (y \geqslant 0) \\ \psi^{(1)}(x,y) = \dfrac{q_{15}^{(1)}\varepsilon_{11}^{(1)} - d_{11}^{(1)}e_{15}^{(1)}}{\varepsilon_{11}^{(1)}\mu_{11}^{(1)} - d_{11}^{(1)2}} w^{(1)}(x,y) + \dfrac{2}{\pi}\displaystyle\int_0^\infty C_1(s)\mathrm{e}^{-sy}\cos(sx)\mathrm{d}s \end{cases} \tag{11}$$

$$\begin{cases} w^{(2)}(x,y) = \frac{2}{\pi}\int_0^\infty A_2(s)\mathrm{e}^{sy}\cos(sx)\mathrm{d}s \\ \phi^{(2)}(x,y) = \dfrac{\mu_{11}^{(2)}e_{15}^{(2)} - d_{11}^{(2)}q_{15}^{(2)}}{\varepsilon_{11}^{(2)}\mu_{11}^{(2)} - d_{11}^{(2)2}} w^{(2)}(x,y) + \dfrac{2}{\pi}\displaystyle\int_0^\infty B_2(s)\mathrm{e}^{sy}\cos(sx)\mathrm{d}s, \quad (y \leqslant 0) \\ \psi^{(2)}(x,y) = \dfrac{q_{15}^{(2)}\varepsilon_{11}^{(2)} - d_{11}^{(2)}e_{15}^{(2)}}{\varepsilon_{11}^{(2)}\mu_{11}^{(2)} - d_{11}^{(2)2}} w^{(2)}(x,y) + \dfrac{2}{\pi}\displaystyle\int_0^\infty C_2(s)\mathrm{e}^{sy}\cos(sx)\mathrm{d}s \end{cases} \tag{12}$$

where $A_1(s)$, $B_1(s)$, $C_1(s)$, $A_2(s)$, $B_2(s)$ and $C_2(s)$ are unknown functions.

So from Eqs. (5)–(7), we have

$$\tau_{yz}^{(1)}(x,y) = -\frac{2}{\pi}\int_0^\infty s\left[\left(c_{44}^{(1)} + \frac{a_1 e_{15}^{(1)}}{a_0} + \frac{a_2 q_{15}^{(1)}}{a_0}\right)A_1(s) + e_{15}^{(1)}B_1(s) + q_{15}^{(1)}C_1(s)\right]\mathrm{e}^{-sy}\cos(sx)\mathrm{d}s \tag{13}$$

$$D_y^{(1)}(x,y) = \frac{2}{\pi}\int_0^\infty s[\varepsilon_{11}^{(1)}B_1(s) + d_{11}^{(1)}C_1(s)]\mathrm{e}^{-sy}\cos(sx)\mathrm{d}s \tag{14}$$

$$B_y^{(1)}(x,y) = \frac{2}{\pi}\int_0^\infty s[d_{11}^{(1)}B_1(s) + \mu_{11}^{(1)}C_1(s)]\mathrm{e}^{-sy}\cos(sx)\mathrm{d}s \tag{15}$$

$$\tau_{yz}^{(2)}(x,y) = \frac{2}{\pi}\int_0^\infty s\left[\left(c_{44}^{(2)} + \frac{g_1 e_{15}^{(2)}}{g_0} + \frac{g_2 q_{15}^{(2)}}{g_0}\right)A_2(s) + e_{15}^{(2)}B_2(s) + q_{15}^{(2)}C_2(s)\right]\mathrm{e}^{sy}\cos(sx)\mathrm{d}s \tag{16}$$

$$D_y^{(2)}(x,y) = -\frac{2}{\pi}\int_0^\infty s[\varepsilon_{11}^{(2)}B_2(s) + d_{11}^{(2)}C_2(s)]\mathrm{e}^{sy}\cos(sx)\mathrm{d}s \tag{17}$$

$$B_y^{(2)}(x,y) = -\frac{2}{\pi}\int_0^\infty s[d_{11}^{(2)}B_2(s) + \mu_{11}^{(2)}C_2(s)]\mathrm{e}^{sy}\cos(sx)\mathrm{d}s \tag{18}$$

where $a_0 = \varepsilon_{11}^{(1)}\mu_{11}^{(1)} - d_{11}^{(1)2}$, $a_1 = \mu_{11}^{(1)}e_{15}^{(1)} - d_{11}^{(1)}q_{15}^{(1)}$, $a_2 = q_{15}^{(1)}\varepsilon_{11}^{(1)} - d_{11}^{(1)}e_{15}^{(1)}$, $g_0 = \varepsilon_{11}^{(2)}\mu_{11}^{(2)} - d_{11}^{(2)2}$, $g_1 = \mu_{11}^{(2)}e_{15}^{(2)} - d_{11}^{(2)}q_{15}^{(2)}$, $g_2 = q_{15}^{(2)}\varepsilon_{11}^{(2)} - d_{11}^{(2)}e_{15}^{(2)}$.

To solve the problem, the jumps of the displacements, the electric and the magnetic potentials across the crack surfaces are defined as follows:

$$f(x) = w^{(1)}(x,0^+) - w^{(2)}(x,0^-) \tag{19}$$

$$f_\phi(x) = \phi^{(1)}(x, 0^+) - \phi^{(2)}(x, 0^-) \tag{20}$$

$$f_\psi(x) = \psi^{(1)}(x, 0^+) - \psi^{(2)}(x, 0^-) \tag{21}$$

Substituting Eqs. (11) and (12) into Eqs. (19)–(21), and applying the Fourier transform and the boundary conditions, it can be obtained

$$\bar{f}(s) = A_1(s) - A_2(s) \tag{21}$$

$$\frac{a_1}{a_0}A_1(s) - \frac{g_1}{g_0}A_2(s) + B_1(s) - B_2(s) = 0 \tag{22}$$

$$\frac{a_2}{a_0}A_1(s) - \frac{g_2}{a_0}A_2(s) + C_1(s) - C_2(s) = 0 \tag{23}$$

Substituting Eqs. (13)–(18) into Eqs. (1)–(3), it can be obtained

$$\left(c_{44}^{(1)} + \frac{a_1 e_{15}^{(1)}}{a_0} + \frac{a_2 q_{15}^{(1)}}{a_0}\right)A_1(s) + e_{15}^{(1)}B_1(s) + q_{15}^{(1)}C_1(s) + \left(c_{44}^{(2)} + \frac{g_1 e_{15}^{(2)}}{g_0} + \frac{g_2 q_{15}^{(2)}}{g_0}\right)A_2(s) + e_{15}^{(2)}B_2(s) + q_{15}^{(2)}C_2(s) = 0 \tag{24}$$

$$\varepsilon_{11}^{(1)}B_1(s) + d_{11}^{(1)}C_1(s) + \varepsilon_{11}^{(2)}B_2(s) + d_{11}^{(2)}C_2(s) = 0 \tag{25}$$

$$d_{11}^{(1)}B_1(s) + \mu_{11}^{(1)}C_1(s) + d_{11}^{(2)}B_2(s) + \mu_{11}^{(2)}C_2(s) = 0 \tag{26}$$

By solving six Eqs. (21)–(26) with six unknown functions $A_1(s)$, $B_1(s)$, $C_1(s)$, $A_2(s)$, $B_2(s)$, $C_2(s)$ and applying the boundary condition (1), it can be obtained:

$$\frac{2\beta_1}{\pi}\int_0^\infty s\bar{f}(s)\cos(sx)\mathrm{d}s = \tau_0, \quad b \leqslant |x| \leqslant 1 \tag{27}$$

$$\int_0^\infty \bar{f}(s)\cos(sx)\mathrm{d}s = 0, \quad |x| < b, \ |x| > 1 \tag{28}$$

where β_1 is a constant which depends on the properties of the materials (see Appendix A). When the properties of the upper and the lower half planes is the same, $\beta_1 = c_{44}^{(1)}/2$. To determine the unknown functions $\bar{f}(s)$, the triple integral equations (27) and (28) must be solved.

3. Solution of the triple Integral equations

From the natural property of the displacement along the crack line, it can be obtained that the jump of the displacements across the crack surface is a finite, continuous and differentiable function. Hence, the jump of the displacements across the crack surfaces can be represented by the following series:

$$f(x)=\sum_{n=0}^{\infty} b_n P_n^{\left(\frac{1}{2},\frac{1}{2}\right)}\left(\frac{x-\frac{1+b}{2}}{\frac{1-b}{2}}\right)\left(1-\frac{\left(x-\frac{1+b}{2}\right)^2}{\left(\frac{1-b}{2}\right)^2}\right)^{\frac{1}{2}}, \quad \text{for } b\leqslant x\leqslant 1 \tag{29}$$

where b_n is unknown coefficients to be determined and $P_n^{(1/2,1/2)}(x)$ is a Jacobi polynomial [17]. The Fourier transform of Eq. (29) are [18]

$$\bar{f}(s)=\sum_{n=0}^{\infty} b_n F_n G_n(s)\frac{1}{s}J_{n+1}\left(s\frac{1-b}{2}\right) \tag{30}$$

$$F_n=2\sqrt{\pi}\frac{\Gamma\left(n+1+\frac{1}{2}\right)}{n!}, \quad G_n(s)=\begin{cases}(-1)^{\frac{n}{2}}\cos\left(s\frac{1+b}{2}\right), & n=0,2,4,6,\ldots\\ (-1)^{\frac{n+1}{2}}\sin\left(s\frac{1+b}{2}\right), & n=1,3,5,7,\ldots\end{cases} \tag{31}$$

where $\Gamma(x)$ and $J_n(x)$ are the Gamma and Bessel functions, respectively.

Substituting Eq. (30) into Eqs. (27) and (28), Eq. (28) has been automatically satisfied. After integration with respect to x in $[b,x](b\leqslant x\leqslant 1)$, Eq. (27) reduces to

$$\sum_{n=0}^{\infty} b_n F_n \int_0^{\infty} s^{-1} G_n(s) J_{n+1}\left(s\frac{1-b}{2}\right)[\sin(sx)-\sin(sb)]\mathrm{d}s=\frac{\pi\tau_0}{2\beta_1}(x-b) \tag{32}$$

From the relationships [17]

$$\int_0^{\infty}\frac{1}{S}J_n(sa)\sin(bs)\mathrm{d}s=\begin{cases}\dfrac{\sin[n\sin^{-1}(b/a)]}{n}, & a>b\\ \dfrac{a^n\sin(n\pi/2)}{n[b+\sqrt{b^2-a^2}]^n}, & a<b\end{cases} \tag{33}$$

$$\int_0^{\infty}\frac{1}{S}J_n(sa)\cos(bs)\mathrm{d}s=\begin{cases}\dfrac{\cos[n\sin^{-1}(b/a)]}{n}, & a>b\\ \dfrac{a^n\cos(n\pi/2)}{n[b+\sqrt{b^2-a^2}]^n}, & a<b\end{cases} \tag{34}$$

The semi-infinite integral in Eq. (32) can be modified as

$$\begin{aligned}&\int_0^{\infty}\frac{1}{s}J_{n+1}\left(s\frac{1-b}{2}\right)\cos\left(s\frac{1+b}{2}\right)\sin(sx)\mathrm{d}s\\ &\quad=\frac{1}{2(n+1)}\left\{\frac{\left(\frac{1-b}{2}\right)^{n+1}\sin\left(\frac{(n+1)\pi}{2}\right)}{\left\{x+\frac{1+b}{2}+\sqrt{\left(x+\frac{1+b}{2}\right)^2-\left(\frac{1-b}{2}\right)^2}\right\}^{n+1}}-\sin\left[(n+1)\sin^{-1}\left(\frac{1+b-2x}{1-b}\right)\right]\right\}\end{aligned} \tag{35}$$

$$\int_0^\infty \frac{1}{s}[1+f(s)]J_{n+1}\left(s\frac{1-b}{2}\right)\sin\left(s\frac{1+b}{2}\right)\sin(sx)\mathrm{d}s$$
$$=\frac{1}{2(n+1)}\left\{\cos\left[(n+1)\sin^{-1}\left(\frac{1+b-2x}{1-b}\right)\right]-\frac{\left(\frac{1-b}{2}\right)^{n+1}\cos\left(\frac{(n+1)\pi}{2}\right)}{\left\{x+\frac{1+b}{2}+\sqrt{\left(x+\frac{1+b}{2}\right)^2-\left(\frac{1-b}{2}\right)^2}\right\}^{n+1}}\right\} \tag{36}$$

Thus the semi-infinite integral in Eq. (32) can be evaluated directly. Eq. (32) can now be solved for the coefficients b_n by the Schmidt method [14,15]. For brevity, the Eq. (32) can be rewritten as following

$$\sum_{n=0}^{\infty} b_n E_n(x) = U(x), \quad b < x < 1 \tag{37}$$

where $E_n(x)$ and $U(x)$ are known functions and coefficients b_n are unknown and will be determined. A set of functions $P_n(x)$ which satisfy the orthogonality conditions

$$\int_b^1 P_m(x)P_n(x)\mathrm{d}x = N_n\delta_{mn}, \quad N_n = \int_b^1 P_n^2(x)\mathrm{d}x \tag{38}$$

can be constructed from the function, $E_n(x)$, such that

$$P_n(x) = \sum_{i=0}^{n} \frac{M_{in}}{M_{nn}} E_i(x) \tag{39}$$

where M_{ij} is the cofactor of the element d_{ij} of D_n, which is defined as

$$D_n = \begin{bmatrix} d_{00}, d_{01}, d_{02}, \ldots, d_{0n} \\ d_{10}, d_{11}, d_{12}, \ldots, d_{1n} \\ d_{20}, d_{21}, d_{22}, \ldots, d_{2n} \\ \cdots \\ \cdots \\ \cdots \\ d_{n0}, d_{n1}, d_{n2}, \ldots, d_{nn} \end{bmatrix}, \quad d_{ij} = \int_b^1 E_i(x)E_j(x)\mathrm{d}x \tag{40}$$

Using Eqs. (37)–(40), we obtain

$$b_n = \sum_{j=n}^{\infty} q_j \frac{M_{nj}}{M_{jj}} \tag{41}$$

$$\text{with } q_j = \frac{1}{N_j}\int_b^1 U(x)P_j(x)\mathrm{d}x \tag{42}$$

4. Intensity factors

The coefficients b_n are known, so that the entire perturbation stress field, the perturbation electric displacement and the magnetic flux can be obtained. However, in fracture mechanics, it is of importance to determine the perturbation stress $\tau_{yz}^{(1)}$, the perturbation electric displacement $D_y^{(1)}$ and the magnetic flux $B_y^{(1)}$ in the vicinity of the crack tips. In the case of the present study, $\tau_{yz}^{(1)}$, $D_y^{(1)}$ and $B_y^{(1)}$ along the crack line can be expressed respectively as

$$\tau_{yz}^{(1)}(x,0) = -\frac{2\beta_1}{\pi}\sum_{n=0}^{\infty} b_n F_n \int_0^{\infty} G_n(s) J_{n+1}\left(s\frac{1-b}{2}\right)\cos(xs)\mathrm{d}s \tag{43}$$

$$D_y^{(1)}(x,0) = -\frac{2\beta_2}{\pi}\sum_{n=0}^{\infty} b_n F_n \int_0^{\infty} G_n(s) J_{n+1}\left(s\frac{1-b}{2}\right)\cos(xs)\mathrm{d}s \tag{44}$$

$$B_y^{(1)}(x,0) = -\frac{2\beta_3}{\pi}\sum_{n=0}^{\infty} b_n F_n \int_0^{\infty} G_n(s) J_{n+1}\left(s\frac{1-b}{2}\right)\cos(xs)\mathrm{d}s \tag{45}$$

where β_2 and β_3 are two constants which depend on the properties of the materials (see Appendix A). When the properties of the upper and the lower half planes is the same, $\beta_2 = e_{15}^{(1)}/2$, $\beta_3 = q_{15}^{(1)}/2$.

Observing the expression in Eqs. (43)–(45), the singular portions of the stress field, the electric displacement and the magnetic flux can be obtained respectively from the relationships [17]

$$\cos\left(s\frac{1+b}{2}\right)\cos(sx) = \frac{1}{2}\left\{\cos\left[s\left(\frac{1+b}{2}-x\right)\right] + \cos\left[s\left(\frac{1+b}{2}+x\right)\right]\right\}$$

$$\sin\left(s\frac{1+b}{2}\right)\cos(sx) = \frac{1}{2}\left\{\sin\left[s\left(\frac{1+b}{2}-x\right)\right] + \sin\left[s\left(\frac{1+b}{2}+x\right)\right]\right\}$$

$$\int_0^{\infty} J_n(sa)\cos(bs)\mathrm{d}s = \begin{cases} \dfrac{\cos[n\sin^{-1}(b/a)]}{\sqrt{a^2-b^2}}, & a>b \\ -\dfrac{a^n \sin(n\pi/2)}{\sqrt{b^2-a^2}[b+\sqrt{b^2-a^2}]^n}, & b>a \end{cases}$$

$$\int_0^{\infty} J_n(sa)\sin(bs)\mathrm{d}s = \begin{cases} \dfrac{\sin[n\sin^{-1}(b/a)]}{\sqrt{a^2-b^2}}, & a>b \\ -\dfrac{a^n \cos(n\pi/2)}{\sqrt{b^2-a^2}[b+\sqrt{b^2-a^2}]^n}, & b>a \end{cases}$$

The singular parts of the stress field, the electric displacement and the magnetic flux can be expressed respectively as follows ($x > 1$ or $x < b$):

$$\tau = -\frac{\beta_1}{\pi}\sum_{n=0}^{\infty} b_n F_n H_n(b,x) \tag{46}$$

$$D = -\frac{\beta_2}{\pi}\sum_{n=0}^{\infty} b_n F_n H_n(b,x) \tag{47}$$

$$B = -\frac{\beta_3}{\pi}\sum_{n=0}^{\infty} b_n F_n H_n(b,x) \tag{48}$$

where

$$H_n(b,x) = \begin{cases} (-1)^{n+1}R(b,x,n), & 0 < x < b \\ -R(b,x,n), & x > 1 \end{cases}$$

$$R(b,x,n) = \frac{2(1-b)^{n+1}}{\sqrt{|1+b-2x|^2-(1-b)^2}\left[|1+b-2x|+\sqrt{|1+b-2x|^2-(1-b)^2}\right]^{n+1}}$$

At the left tip of the right crack, we obtain the stress intensity factor K_{L} as

$$K_{\mathrm{L}} = \lim_{x\to b^-}\sqrt{2\pi(b-x)}\cdot\tau = \beta_1\sqrt{\frac{1}{2\pi(1-b)}}\sum_{n=0}^{\infty}(-1)^n b_n F_n \tag{49}$$

At the right tip of the right crack, we obtain the stress intensity factor K_{R}

$$K_{\mathrm{R}} = \lim_{x\to 1^+}\sqrt{2\pi(x-1)}\cdot\tau = \beta_1\sqrt{\frac{1}{2\pi(1-b)}}\sum_{n=0}^{\infty} b_n F_n \tag{50}$$

At the left tip of the right crack, we obtain the electric displacement intensity factor $K_{\mathrm{L}}^{\mathrm{D}}$ as

$$K_{\mathrm{L}}^{\mathrm{D}} = \lim_{x\to b^-}\sqrt{2\pi(b-x)}\cdot D = \beta_2\sqrt{\frac{1}{2\pi(1-b)}}\sum_{n=0}^{\infty}(-1)^n b_n F_n = \frac{\beta_2}{\beta_1}K_{\mathrm{L}} \tag{51}$$

At the right tip of the right crack, we obtain the electric displacement intensity factor $K_{\mathrm{R}}^{\mathrm{D}}$ as

$$K_{\mathrm{R}}^{\mathrm{D}} = \lim_{x\to 1^+}\sqrt{2\pi(x-1)}\cdot D = \beta_2\sqrt{\frac{1}{2\pi(1-b)}}\sum_{n=0}^{\infty} b_n F_n = \frac{\beta_2}{\beta_1}K_{\mathrm{R}} \tag{52}$$

At the left tip of the right crack, we obtain the magnetic intensity factor $K_{\rm L}^{\rm B}$ as

$$K_{\rm L}^{\rm B} = \lim_{x\to b^-} \sqrt{2\pi(b-x)}\cdot B = \beta_3\sqrt{\frac{1}{2\pi(1-b)}}\sum_{n=0}^{\infty}(-1)^n b_n F_n = \frac{\beta_3}{\beta_1}K_{\rm L} \tag{53}$$

At the right tip of the right crack, we obtain the magnetic intensity factor $K_{\rm R}^{\rm B}$ as

$$K_{\rm R}^{\rm B} = \lim_{x\to 1^+} \sqrt{2\pi(x-1)}\cdot D = \beta_3\sqrt{\frac{1}{2\pi(1-b)}}\sum_{n=0}^{\infty} b_n F_n = \frac{\beta_3}{\beta_1}K_{\rm R} \tag{54}$$

5. Conclusions

In the present paper,it is assumed that two collinear interface cracks only subject to an anti-plane shear stress loading, do not subject to an electric field or a magnetic flux loading. Certainly, the loading and the geometry of cracks are symmetry. In this case, the results of this paper are shown in Fig. 2. From Eqs. (32), (49) and (50), it can be obtained that the stress field does not depend on the material properties except the crack length. So in all computation, the material constants are not considered. From the results, the following observations are very significant:

(i) The properties of the stress intensity factor of the present paper are the same as ones in a general elastic material. The stress intensity factor does not depend on the properties of the material for the anti-plane shear fracture problem. This can be obtained in Eqs. (32), (49) and (50). However, the electric displacement and the magnetic flux intensity factors depend on the properties of the materials as shown in Eqs. (51)–(54).

(ii) The stress intensity factors decrease with increase in the distance between two cracks. It can be also obtained that the interaction of two interface cracks decrease with increase in the distance between two cracks. The results of the electric displacement and the magnetic flux intensity factors can be obtained through the Eqs. (51)–(54). In the present paper, they are omitted. Hence, the electric displacement and the magnetic flux intensity factors have the same changing rule as the stress intensity factor.

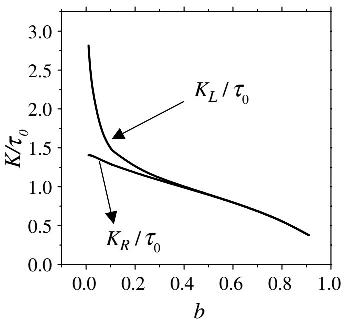

Fig. 2. The stress intensity factor versus b.

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Appendix A

$$H_1 = -2d_{11}^{(1)}e_{15}^{(1)}c_{44}^{(2)}q_{15}^{(1)} + e_{15}^{(2)^2}q_{15}^{(1)^2} + c_{44}^{(2)}\varepsilon_{11}^{(1)}q_{15}^{(1)^2} - 2e_{15}^{(1)}e_{15}^{(2)}q_{15}^{(1)}q_{15}^{(2)} + e_{15}^{(1)^2}q_{15}^{(2)^2} - 2e_{15}^{(1)}c_{44}^{(2)}q_{15}^{(1)}d_{11}^{(2)}$$

$$H_2 = c_{44}^{(2)}q_{15}^{(1)^2}\varepsilon_{11}^{(2)} + e_{15}^{(1)^2}c_{44}^{(2)}\left(\mu_{11}^{(2)} + \mu_{11}^{(1)}\right)$$

$$H_3 = c_{44}^{(1)}\left[-d_{11}^{(1)^2}c_{44}^{(2)} - 2e_{15}^{(2)}q_{15}^{(2)}d_{11}^{(2)} - c_{44}^{(2)}d_{11}^{(2)^2} - 2d_{11}^{(1)}\left(e_{15}^{(2)}q_{15}^{(2)} + c_{44}^{(2)}d_{11}^{(2)}\right) + q_{15}^{(2)^2}\varepsilon_{11}^{(2)}\right]$$

$$H_4 = c_{44}^{(1)}\left\{e_{15}^{(2)^2}\mu_{11}^{(2)} + c_{44}^{(2)}\varepsilon_{11}^{(2)}\mu_{11}^{(2)} + e_{15}^{(2)^2}\mu_{11}^{(1)} + c_{44}^{(2)}\varepsilon_{11}^{(2)}\mu_{11}^{(1)} + \varepsilon_{11}^{(1)}\left[q_{15}^{(2)^2} + c_{44}^{(2)}\left(\mu_{11}^{(2)} + \mu_{11}^{(1)}\right)\right]\right\}$$

$$R_1 = -d_{11}^{(1)^2}c_{44}^{(2)} + \varepsilon_{11}^{(1)}q_{15}^{(1)^2} + 2\varepsilon_{11}^{(1)}q_{15}^{(1)}q_{15}^{(2)} + \varepsilon_{11}^{(1)}q_{15}^{(2)^2} - 2e_{15}^{(1)}q_{15}^{(1)}d_{11}^{(2)} - 2e_{15}^{(2)}q_{15}^{(1)}d_{11}^{(2)}$$

$$R_2 = -2e_{15}^{(1)}q_{15}^{(2)}d_{11}^{(2)} - 2e_{15}^{(2)}q_{15}^{(2)}d_{11}^{(2)} - c_{44}^{(2)}d_{11}^{(2)^2}$$

$$R_3 = -2d_{11}^{(1)}\left[e_{15}^{(1)}\left(q_{15}^{(1)} + q_{15}^{(2)}\right) + e_{15}^{(2)}\left(q_{15}^{(1)} + q_{15}^{(2)}\right) + c_{44}^{(2)}d_{11}^{(2)}\right]$$

$$R_4 = q_{15}^{(1)^2}\varepsilon_{11}^{(2)} + 2q_{15}^{(1)}q_{15}^{(2)}\varepsilon_{11}^{(2)} + q_{15}^{(2)^2}\varepsilon_{11}^{(2)} + e_{15}^{(1)^2}\mu_{11}^{(2)} + 2e_{15}^{(1)}e_{15}^{(2)}\mu_{11}^{(2)} + e_{15}^{(2)^2}\mu_{11}^{(2)} + c_{44}^{(2)}\varepsilon_{11}^{(1)}\mu_{11}^{(2)}$$

$$R_5 = c_{44}^{(2)}\varepsilon_{11}^{(2)}\mu_{11}^{(2)} + e_{15}^{(1)^2}\mu_{11}^{(1)} + 2e_{15}^{(1)}e_{15}^{(2)}\mu_{11}^{(1)} + e_{15}^{(2)^2}\mu_{11}^{(1)} + c_{44}^{(2)}\varepsilon_{11}^{(1)}\mu_{11}^{(1)} + c_{44}^{(2)}\varepsilon_{11}^{(2)}\mu_{11}^{(1)}$$

$$R_6 = -c_{44}^{(1)}\left[d_{11}^{(1)^2} + 2d_{11}^{(1)}d_{11}^{(2)} + d_{11}^{(2)^2} - \left(\varepsilon_{11}^{(1)} + \varepsilon_{11}^{(2)}\right)\left(\mu_{11}^{(1)} + \mu_{11}^{(2)}\right)\right]$$

$$\beta_1 = (H_1 + H_2 + H_3 + H_4)/(R_1 + R_2 + R_3 + R_4 + R_5 + R_6)$$

$$S_1 = -e_{15}^{(2)}\varepsilon_{11}^{(1)}q_{15}^{(1)^2} - e_{15}^{(2)}\varepsilon_{11}^{(1)}q_{15}^{(1)}q_{15}^{(2)} + e_{15}^{(1)}e_{15}^{(2)}q_{15}^{(1)}d_{11}^{(2)} - c_{44}^{(2)}\varepsilon_{11}^{(1)}q_{15}^{(1)}d_{11}^{(2)} + e_{15}^{(1)^2}q_{15}^{(2)}d_{11}^{(2)}$$

$$S_2 = 2e_{15}^{(1)}e_{15}^{(2)}q_{15}^{(2)}d_{11}^{(2)} + e_{15}^{(1)}c_{44}^{(2)}d_{11}^{(2)^2} - e_{15}^{(1)}q_{15}^{(1)}q_{15}^{(2)}\varepsilon_{11}^{(2)} - e_{15}^{(1)}q_{15}^{(2)^2}\varepsilon_{11}^{(2)}$$

$$S_3 = d_{11}^{(1)}\left\{e_{15}^{(1)}\left[e_{15}^{(2)}\left(2q_{15}^{(1)} + q_{15}^{(2)}\right) + c_{44}^{(2)}d_{11}^{(2)}\right] + q_{15}^{(1)}\left(e_{15}^{(2)^2} + c_{44}^{(2)}\varepsilon_{11}^{(2)}\right)\right\}$$

$$S_4 = -e_{15}^{(1)^2}e_{15}^{(2)}\mu_{11}^{(2)} - e_{15}^{(1)}e_{15}^{(2)^2}\mu_{11}^{(2)} - e_{15}^{(1)}c_{44}^{(2)}\varepsilon_{11}^{(2)}\mu_{11}^{(2)} - e_{15}^{(1)^2}e_{15}^{(2)}\mu_{11}^{(1)} - e_{15}^{(1)}e_{15}^{(2)^2}\mu_{11}^{(1)} - e_{15}^{(1)}c_{44}^{(2)}\varepsilon_{11}^{(2)}\mu_{11}^{(1)}$$

$$S_5 = c_{44}^{(1)}\left\{d_{11}^{(1)^2}e_{15}^{(2)} + d_{11}^{(1)}\left(e_{15}^{(2)}d_{11}^{(2)} - q_{15}^{(2)}\varepsilon_{11}^{(2)}\right) + \varepsilon_{11}^{(1)}\left[q_{15}^{(2)}d_{11}^{(2)} - e_{15}^{(2)}\left(\mu_{11}^{(2)} + \mu_{11}^{(1)}\right)\right]\right\}$$

$$\beta_2 = -(S_1 + S_2 + S_3 + S_4 + S_5)/(R_1 + R_2 + R_3 + R_4 + R_5 + R_6)$$

$$Y_1 = \varepsilon_{11}^{(1)} q_{15}^{(1)^2} q_{15}^{(2)} + \varepsilon_{11}^{(1)} q_{15}^{(1)} q_{15}^{(2)^2} - e_{15}^{(2)} q_{15}^{(1)^2} d_{11}^{(2)} - e_{15}^{(1)} q_{15}^{(1)} q_{15}^{(2)} d_{11}^{(2)} - 2e_{15}^{(2)} q_{15}^{(1)} q_{15}^{(2)} d_{11}^{(2)} - c_{44}^{(2)} q_{15}^{(1)} d_{11}^{(1)^2}$$

$$Y_2 = q_{15}^{(1)^2} q_{15}^{(2)} \varepsilon_{11}^{(2)} + q_{15}^{(1)} q_{15}^{(2)^2} \varepsilon_{11}^{(2)} + e_{15}^{(1)} e_{15}^{(2)} q_{15}^{(1)} \mu_{11}^{(2)} + e_{15}^{(2)^2} q_{15}^{(1)} \mu_{11}^{(2)} + c_{44}^{(2)} \varepsilon_{11}^{(1)} q_{15}^{(1)} \mu_{11}^{(2)} + c_{44}^{(2)} q_{15}^{(1)} \varepsilon_{11}^{(2)} \mu_{11}^{(2)}$$

$$Y_3 = -d_{11}^{(1)} \left(2e_{15}^{(1)} q_{15}^{(1)} q_{15}^{(2)} + e_{15}^{(2)} q_{15}^{(1)} q_{15}^{(2)} + e_{15}^{(1)} q_{15}^{(2)^2} + c_{44}^{(2)} q_{15}^{(1)} d_{11}^{(2)} + c_{44}^{(2)} e_{15}^{(1)} \mu_{11}^{(2)} \right)$$

$$Y_4 = e_{15}^{(1)^2} q_{15}^{(2)} \mu_{11}^{(1)} + e_{15}^{(1)} e_{15}^{(2)} q_{15}^{(2)} \mu_{11}^{(1)} + e_{15}^{(1)} c_{44}^{(2)} d_{11}^{(2)} \mu_{11}^{(1)}$$

$$Y_5 = c_{44}^{(1)} \left(- d_{11}^{(1)^2} q_{15}^{(2)} - d_{11}^{(1)} d_{11}^{(2)} q_{15}^{(2)} + d_{11}^{(1)} e_{15}^{(2)} \mu_{11}^{(2)} + \varepsilon_{11}^{(1)} q_{15}^{(2)} \mu_{11}^{(1)} - e_{15}^{(1)} d_{11}^{(2)} \mu_{11}^{(1)} + q_{15}^{(2)} \varepsilon_{11}^{(2)} \mu_{11}^{(1)} \right)$$

$$\beta_3 = (Y_1 + Y_2 + Y_3 + Y_4 + Y_5)/(R_1 + R_2 + R_3 + R_4 + R_5 + R_6)$$

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